Considerations of Material Behaviour in the Numerical Solution of Cyclic Thermal an Mechanical Loading using Kinematic Hardening

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Introduction

If a cyclic force-controlled loading is applied to a system such that plastic deformations and redeformations appear there can be an increase of the plastic strain each cycle. In a displacement-controlled experiment this results in a moving of the mean stress.
If the extreme values of the plastic strain come to a standstill this effect is called shakedown. If the changes in strain are only elastic at that point it is called elastic shakedown, if a hysteresis remains it is called plastic shakedown.
Under some circumstances it is considered that the extreme values of the plastic strain grow with a constant increment each cycle. This effect is called ratchetting. Ratchetting appears in isothermal as well as non-isothermal cases, and one has to distinguish between material and structural ratchetting. In the isothermal case ratchetting normally can appear if the mean stress of the cyclic loading differs from 0. In this paper we only consider material effects in thermal ratchetting where stress and temperature change in the same rhythm.

The application where this problem occurred was a pipe in a power plant containing once hot, once cold water (insurge/outsurge). The structural aspects due to the temperature gradients and different phases in the hot state are excluded here. We focus on the fact that due to constraints the thermal expansion leads to stresses higher than the yield stress in both heating as well as cooling.

This paper discusses what is calculated by the ANSYS plasticity models.

Kinematic hardening

For better understanding we consider stress-controlled uniaxial cases. The yield condition for the kinematic hardening reads

$$J(\sigma-\alpha)\leq\sigma_y \qquad (1)$$

where J denotes the von Mises condition, σ the total stresses, α the back stresses, the deviation of the yield surface, and σ_y the initial yield stress.
α is measured in stress units. Therefore, the question arises what happens when the stress-strain relation changes due to a change in temperature. The US-Nuclear Standard NE F 9-5T [1] tells us that a temperature-dependent change of α should be

taken into account, but it does not say how.

Bilinear kinematic hardening

For the bilinear kinematic hardening (BKIN) the ANSYS Theoretical Manual [2] tells us that the increment of the back stresses is determined by

$$d\alpha = C d\varepsilon^{pl} \tag{2}$$

where ε^{pl} is the plastic strain, C the hardening modulus

$$C = \left(\frac{2}{3}\right)\frac{E E_T}{E - E_T} \tag{3}$$

E is Young's modulus and E_T the tangential modulus. As far as C is constant there is a unique relation between ε^{pl} and α (Fig. 1).

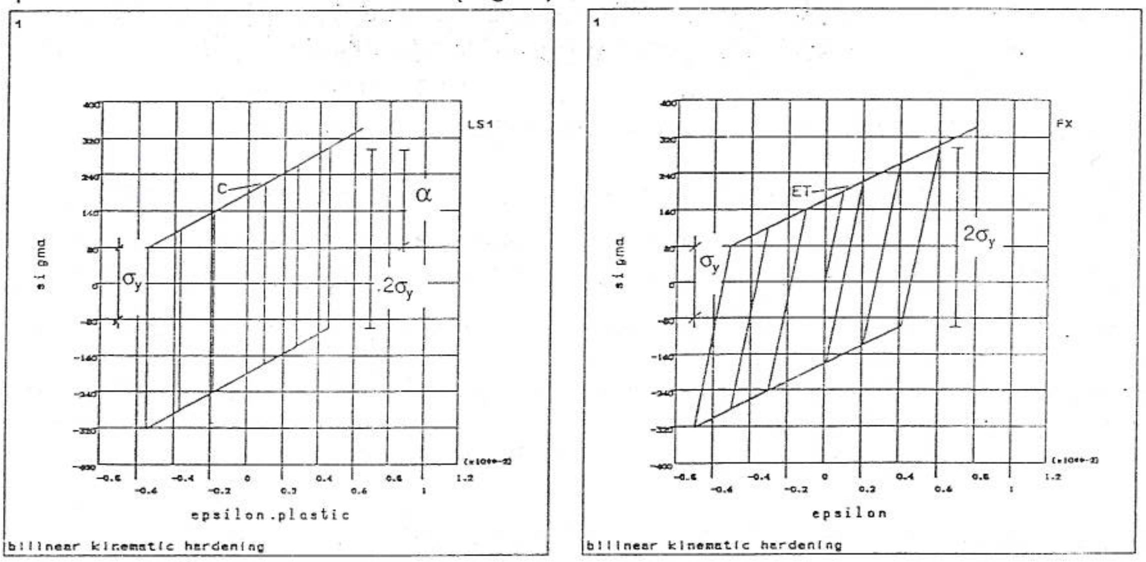

Fig. 1: Bilinear kinematic hardening, isothermal cycles with different load levels

The bilinear model is a first approximation for the Bauschinger effect. In the isothermal case no material ratchetting can be determined.

As to be seen in the sample of USERPL and to be read in QA notice 93-10 [3] BKIN is not implemented as given by eq. (1) and (2) but follows the O.R.N.L. algorithm. In each load step $d\alpha$ is divided by 2G, G being the shear modulus, to obtain an equivalent strain

$$d\varepsilon^{shift} = \frac{1}{2G} d\alpha = \frac{C}{2G} d\varepsilon^{pl} \tag{4}$$

1/2G is used because the plastic strain increments are deviatoric. With **D** being the elastic constitutive matrix, $\mathbf{D}^{-1}$ or 1/E in one dimension would lead to the same result. The yield condition then reads

$$J(D(\varepsilon^{el} - \varepsilon^{shift})) \leq \sigma_y \tag{5}$$

The back stresses at the actual temperature T_{act} are

$$\alpha(T_{act}) = D(T_{act}) \int D(T)^{-1} C(T) d\varepsilon^{pl} = D(T_{act}) \int D(T(t))^{-1} C(T(t)) \dot{\varepsilon}^{pl} dt \tag{6}$$

where the temperature T varies during the loading history, i.e. the temperature dependency of α is related to that of Young's modulus E. There is no difference between eq. (1,2) and (4,5) if E does not depend on the temperature but C resp. E_T and σ_y do (Fig. 3).

Now consider the stress-strain curve in fig. 3 as a result of the temperature and load history in fig. 2 (loading, unloading at T1, temperature change, reloading, unloading at T2).

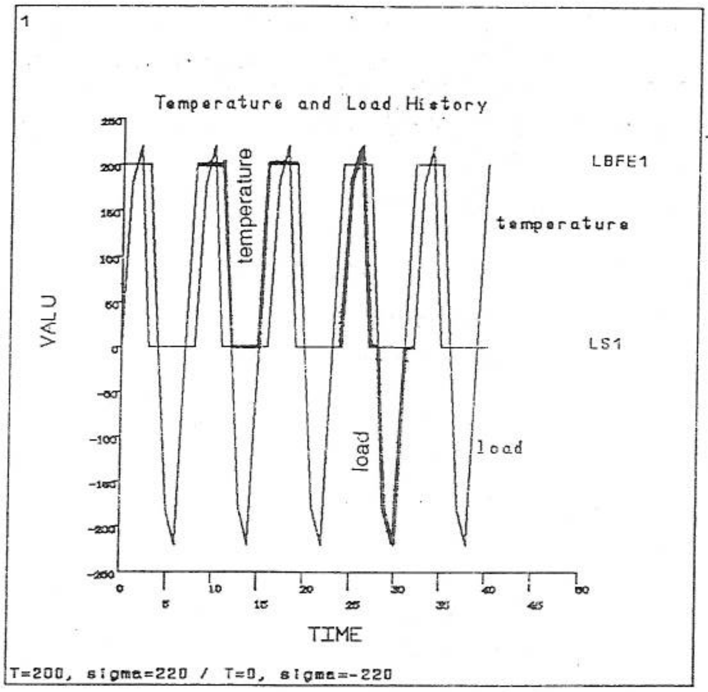

Fig. 2: Temperature and load history

The amount of ε^{pl} is different for both loading directions although the absolute maximum stresses are the same. Therefore, yield appears earlier and the original curve is not matched. This effect is repeated in each cycle - thermal ratchetting is calculated.

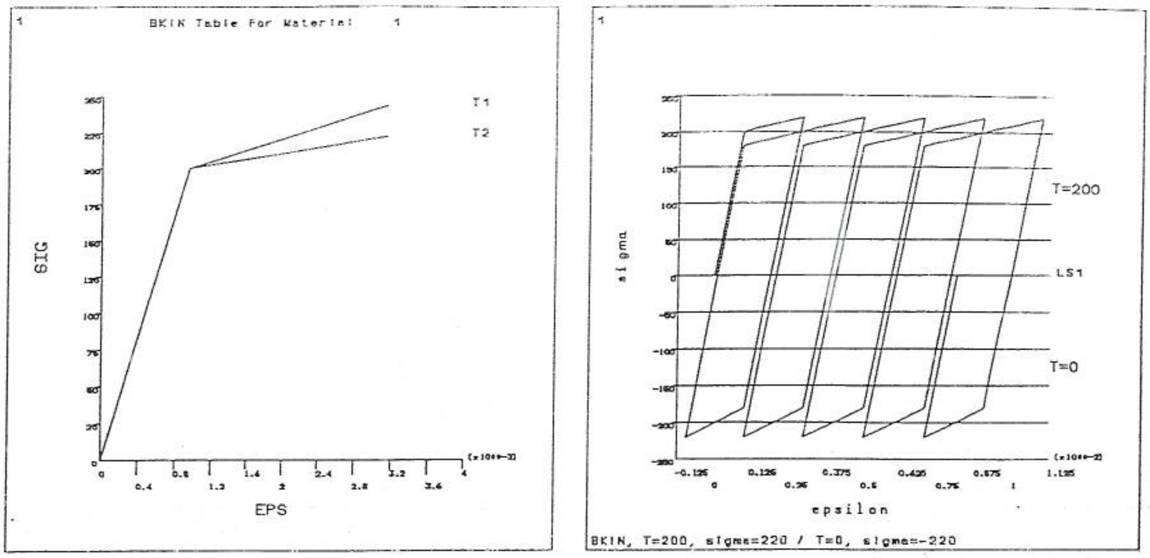

Fig. 3: Standard BKIN model

Now consider the case of similar material curves for different temperatures T_1 and T_2, i.e. the curves differ from each other only by one factor. In the bilinear case this means

$$\left.\frac{E}{E_T}\right|_{T_1} = \left.\frac{E}{E_T}\right|_{T_2} \rightarrow \left.\frac{E}{C}\right|_{T_1} = \left.\frac{E}{C}\right|_{T_2} \qquad (7)$$

(Fig. 4,5). Now the ratchetting vanishes because α is recalculated by eq. (6) such that the original curve is matched.

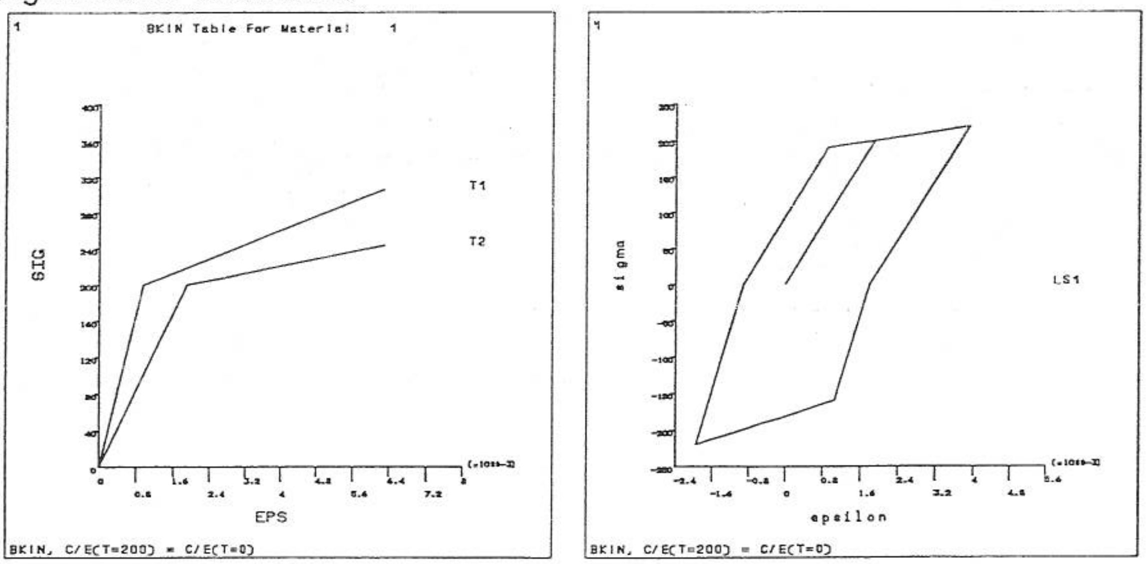

Fig. 4: BKIN, same temperature-dependency for E and C

The ratchetting phenomenon here is sensitive to the value of E. However, for austenitic steel the exact value of E depends on the engineer's estimate of the slope of the stress-strain relation. Therefore, the ANSYS choice for the temperature dependency of α is not very reasonable, because the ratchetting - a phenomenon of plasticity - depends on that definition of the elastic properties.

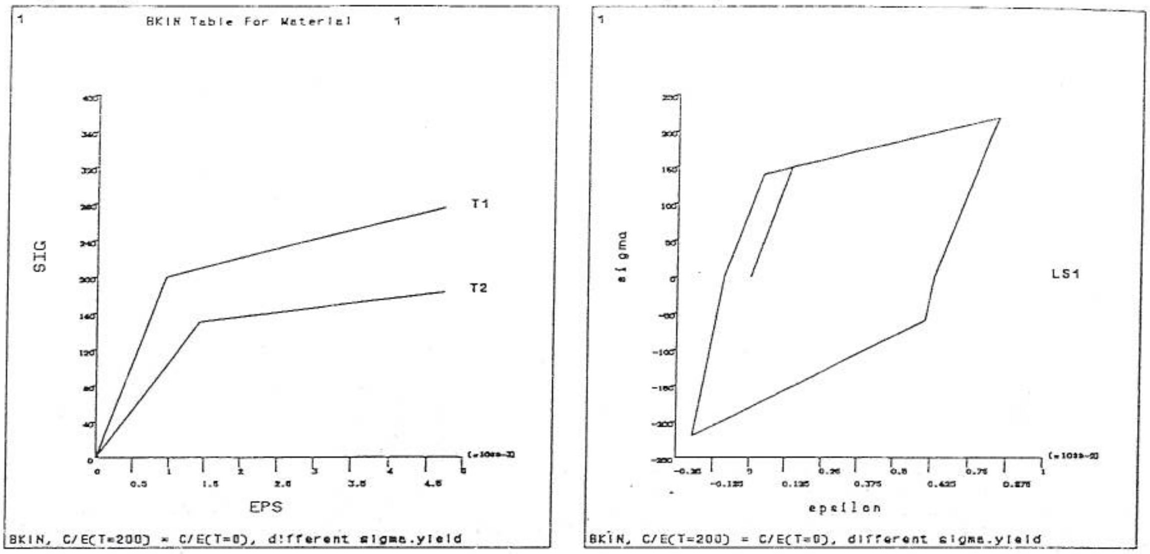

Fig. 5: BKIN, similar stress-strain curves

Putting eq. (7) into (8) one obtains

$$\alpha = C\int d\varepsilon^{pl} = C\varepsilon^{pl} \tag{8}$$

because $C(T_{act})/E(T_{act}) = C(T)/E(T) = const.$
This means α is independent of the temperature history. Several authors[e.g.4,6] agree with that postulate but some with doubts. This assumption is not derived from a universal natural principle, it is just a postulate.

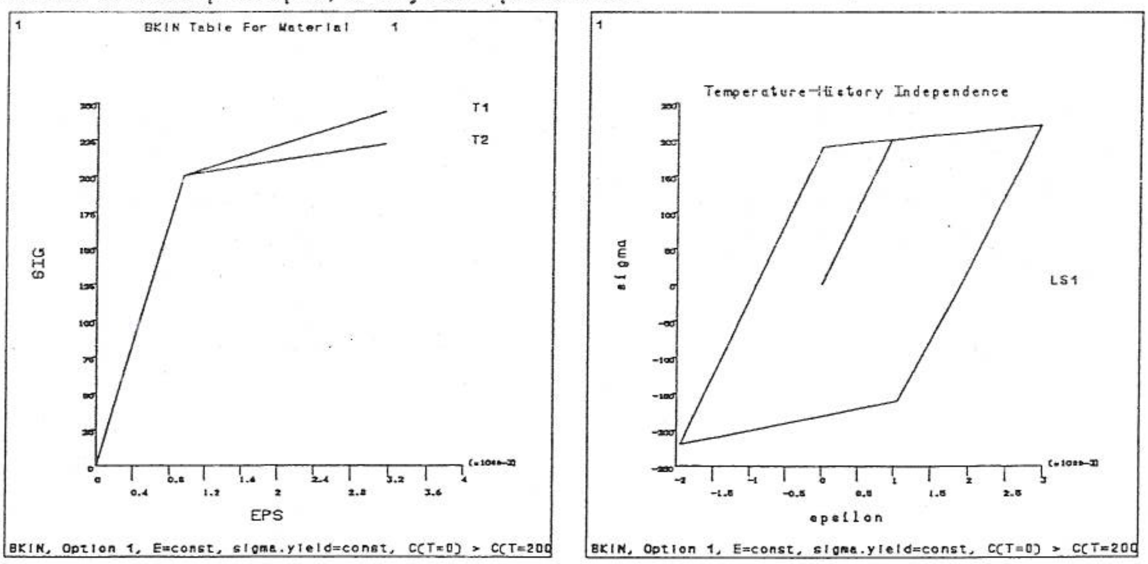

Fig. 6: BKIN, temperature-history independent back stresses

For arbitrary relations between C and E eq. (8) can easily be implemented or - in order to be compatible with the standard programming -

$$\varepsilon^{shift} = \left.\frac{C}{2G}\right|_{T_{act}} \varepsilon^{pl} \tag{9}$$

can be used. Rice [7] is cited in conjunction with this form but he does not explicitly write that.

With this modification (TB,BKIN,,,,1 in Rev. 5.0A+) the result of fig. 6 is obtained showing no ratchetting effect. With this modification material ratchetting in one dimension cannot be obtained under any circumstances. Wether this agrees with physics cannot be answered because no technical material has real linear hardening behaviour.

Taking the total differential of (8) one obtains

$$d\alpha = C d\varepsilon^{pl} + \varepsilon^{pl}\frac{\partial C}{\partial T}dT \tag{10}$$

Therefore, a German author[8] who strongly believes to eq. (8) states that a term is missing in ANSYS. However, the total differential of α must read

$$d\alpha = \frac{\partial \alpha}{\partial \varepsilon^{pl}}d\varepsilon^{pl} + \frac{\partial \alpha}{\partial T}dT \tag{11}$$

The models above differ in the choice of $\partial\alpha/\partial T$. This is only a choice, although $\partial\alpha/\partial T \sim \partial C/\partial T$ seems to be more reasonable than $\partial\alpha/\partial T \sim \partial E/\partial T$ (Standard-ANSYS) or even $\partial\alpha/\partial T = 0$ (Theoretical Manual).

Multilinear kinematic hardening

The piecewise linear kinematic hardening model in ANSYS (MKIN) is coded according to Besseling[9]. In the one-dimensional case it can be represented by elastic-ideally plastic spars where the sum over the cross section areas equals the total area. The single areas or more general the weighting factors t_i for the spring stiffness are chosen such that the reduction of the tangent modulus is t_iE when the i-th spar begins yielding. Instead of different cross sections different Young's moduli can be used. The state variables are the plastic strains of the spars. Although there is no α in this concept the multilinear model behaves like the bilinear one as far as they are comparable. There is no thermal ratchetting when the material curves are similar.
Instead of using MKIN one can discretize the parallel spars where the material model is BKIN or BISO with E_T=0. This system behaves like a single spar with MKIN as long as the temperature does not change. In the non-isothermal case, however, differences appear. For the material curves of fig. 7 and the load and temperature history of fig. 2 the stress-strain curves are shown in fig. 7 (MKIN) and 8 (parallel spars). While the parallel spar model shows no ratchetting, with MKIN increasing plastic strain occurs. However it is not a constant repetition like in the standard bilinear model, but convergence fails within the fourth cycle. Under certain

circumstances increasing strain increments can appear.

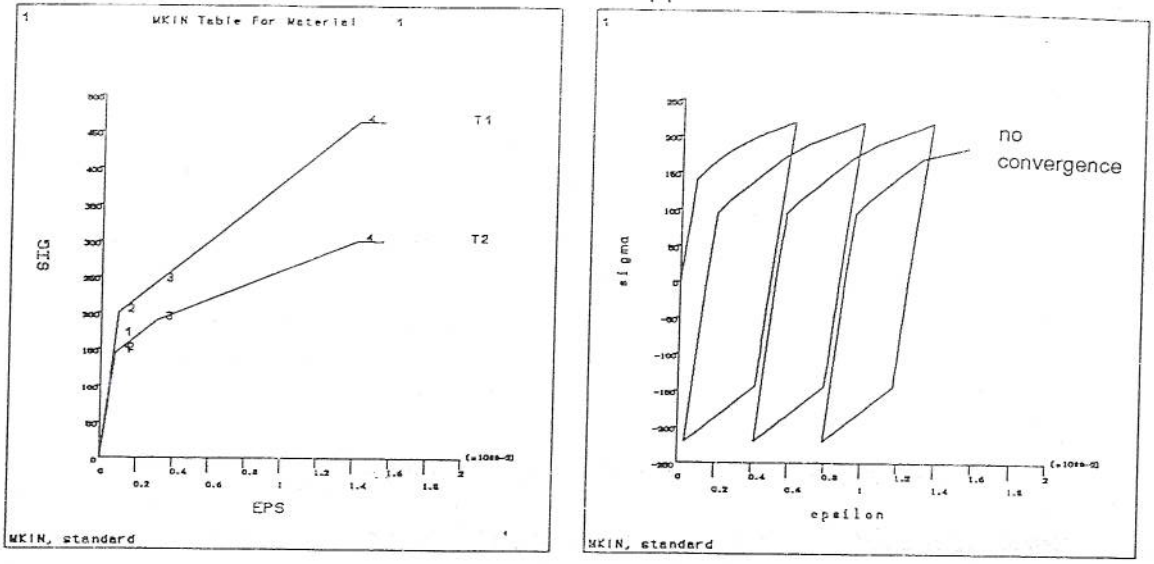

Fig. 7: Standard MKIN, ratchetting and unstable cycles

Temp.	0		200	
Spar	E^i	σ_y^i	E^i	σ_y^i
1	0	0	$1.6 \cdot 10^5$	128
2	$1.9 \cdot 10^5$	180.95	0	0
3	0	0	$0.1 \cdot 10^5$	31
4	$0.2 \cdot 10^5$	282	$0.1 \cdot 10^5$	141
Σ	$2.1 \cdot 10^5$		$1.8 \cdot 10^5$	

Table 1: Parameters for the parallel spar model

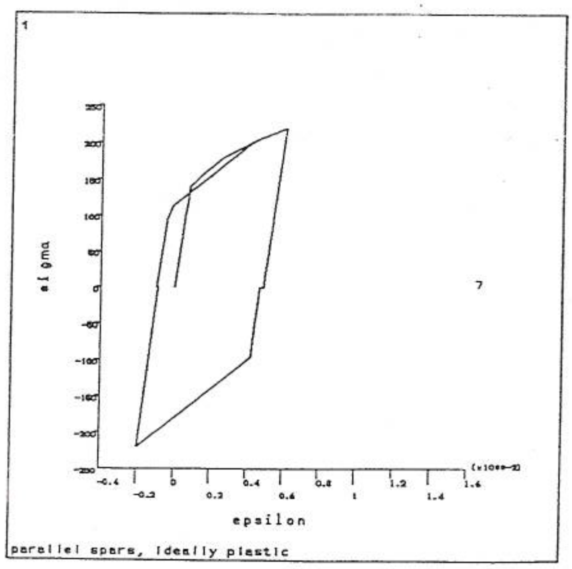

Fig. 8: Behaviour of the discretized parallel spar model

The reason is the calculation of plastic strains. For each sublayer (generalized expression for the spars in MKIN) a plastic strain increment is calculated. The total plastic strain increment then is the weighted sum over all sublayers:

$$\Delta \varepsilon^{pl} = \sum_i t_i \Delta \varepsilon_i^{pl} \tag{12}$$

In the isothermal case this results in

$$\varepsilon^{pl} = \sum_i t_i \varepsilon_i^{pl} \tag{13}$$

because the t_i are constant. This does not hold in the non-isothermal case, where the t_i are temperature-dependent. However, eq. (13) should be valid in all cases to comply with the parallel spar model. By recalculating ε^{pl} from ε_i^{pl} and t_i in each load increment the difference vanishes. If similar material curves are used only E changes with temperature whereas the t_i remain constant. Therefore, no problem arises.

Besseling did not discuss the non-isothermal case, i.e. MKIN is a correct implementation of the Besseling model from this point of view, but it does not make sense when the temperature changes. Only the small modification (13) is needed to obtain a "consistent Besseling model" which behaves like the underlying image of the parallel spars and can be applied to the non-isothermal case.

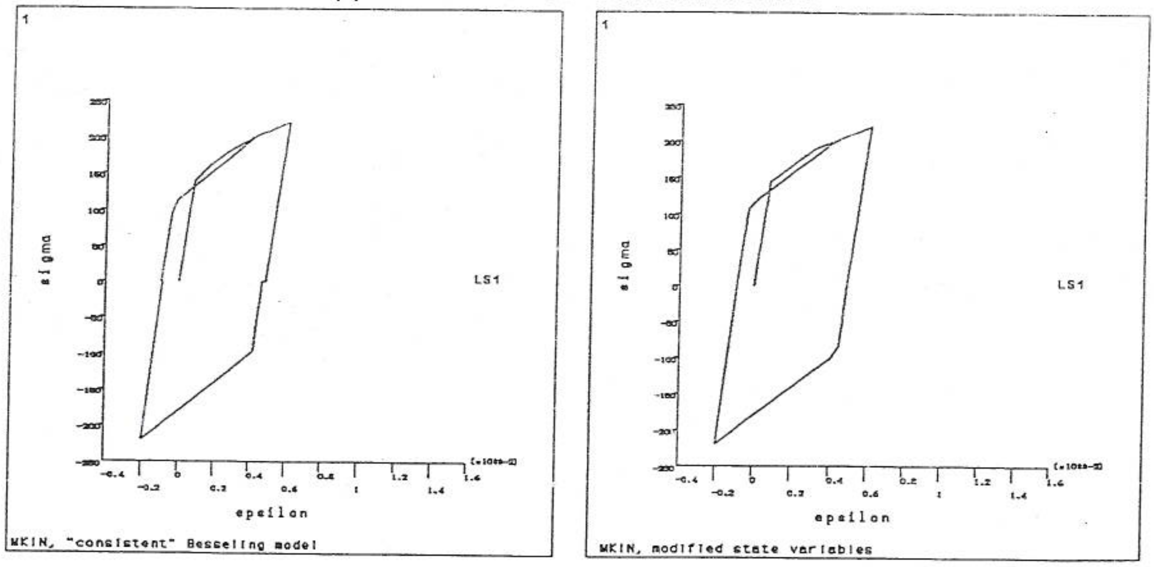

Fig. 9: MKIN a) Consistent Besseling model, recalculated plastic strain
b) Scaled state variables

With the load- and temperature-history of fig. 2 the result of fig. 9a shows strain changes when only the temperature changes, because the plastic strain is recalculated. This is probably not according to experiments. The result can be isothermally obtained by applying the same total strain, not the plastic strain like in the temperature-history independent model. An alternative is to scale the state variables (one factor c_{scale} for each component of ε^{pl}) such that

$$\sum_i t_i \varepsilon_i^{pl} c_{scale} = \varepsilon_{tot}^{pl} \quad (14)$$

holds. The strain changes at load level 0 vanish (fig. 9b)

Relations between isothermal and non-isothermal cycles

The postulate of the temperature-history independence means that α should be calculated as if the plastic strain is reached at a constant temperature. To compare with the non-isothermal behaviour the extreme value of stress in one loading direction must be modified in the isothermal case. This means non-isothermal loading with mean stress 0 is equivalent to an isothermal cycle with a mean stress differing from 0. This makes a thermally induced ratchetting more probable.

Chaboche model

The kinematic hardening models are not appropriate tools to predict ratchetting and shakedown. Therefore, a Chaboche[4,5] model has been taken under consideration and coded as USERPL. It is characterized by the combination of isotropic and kinematic hardening in the yield condition

$$J(\sigma-\alpha) \leq \sigma_y + R \quad (15)$$

and the evolution equations

$$d\alpha = \sum_{i=1}^{m} (C_i d\varepsilon^{pl} - \gamma_i \alpha_i dp) \quad (16)$$

$$dR = b(Q-R)dp \quad (17)$$

where C_i,γ_i,b and Q are material parameters, dp the equivalent plastic strain increment

$$dp = \sqrt{d\varepsilon_{ij}^{pl} d\varepsilon_{ij}^{pl}} \quad (18)$$

If m=1, C denotes the initial hardening modulus and C/γ the saturation value for α. Q is the saturation value for R.
Eq. (16) can be written as

$$d\alpha = (C\mathbf{N} - \gamma\alpha)dp \quad (19)$$

where **N** denotes the direction of the plastic strain which is

$$\mathbf{N} = \frac{\partial f/\partial\sigma}{\|\partial f/\partial\sigma\|} \quad (20)$$

according to Drucker's postulate. We get the differential equation

$$\alpha'(p) + \gamma\alpha(p) = C\mathbf{N} \quad (21)$$

with the boundary condition

$$\alpha(p{=}p_0) = \alpha_0 \qquad (22)$$

This equation can be solved as long as **N** is constant, i.e. at least during a load step. p_0 and α_0 are cumulated plastic strain and back stresses at the beginning of the load increment. With

$$\alpha = \alpha_0 + \Delta\alpha \ , \quad p = p_0 + \Delta p \qquad (23)$$

we can replace α' by $d\Delta\alpha/d\Delta p$ and rewrite

$$\begin{aligned} \Delta\alpha' + \gamma(\alpha_0 + \Delta\alpha) &= CN \\ \rightarrow \Delta\alpha' + \gamma\Delta\alpha &= CN - \gamma\alpha_0 \\ b.c.{:}\ \Delta\alpha(\Delta p{=}0) &= 0 \end{aligned} \qquad (24)$$

which results in

$$\alpha = \alpha_0 + \left(\frac{CN}{\gamma} - \alpha_0\right)\left(1 - e^{-\gamma(p-p_0)}\right) \qquad (25)$$

Q may depend on p:

$$Q = Q_M + (Q_0 - Q_M)e^{-2\mu p} \qquad (26)$$

Q_M,Q_0 and μ are additional material parameters. Eq. (17) describes the short-range behaviour, (26) the long-range terms.
For the following examples $E=2.1\cdot10^5$, $C=3\cdot10^4$ and $\sigma_y=200$ are used at T=0 and $E=1.8\cdot10^5$, $C=5.5\cdot10^4$ and $\sigma_y=144$ at T=200 ($\gamma=700$, b=Q=0). Fig 10 shows stress-strain curves for the two temperatures.

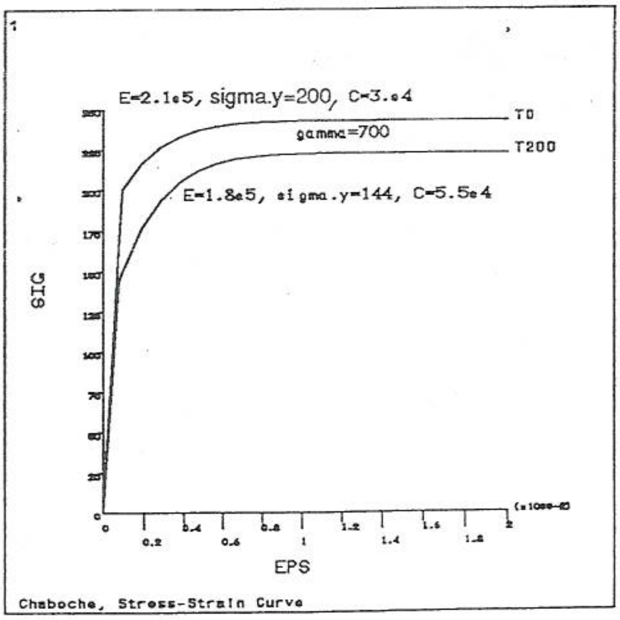

Fig. 10: Chaboche model, stress-strain curves

Fig. 11a and b show the behaviour with constant temperature T=200 resp. T=0. It looks like the typical kinematic hardening including the Bauschinger effect.

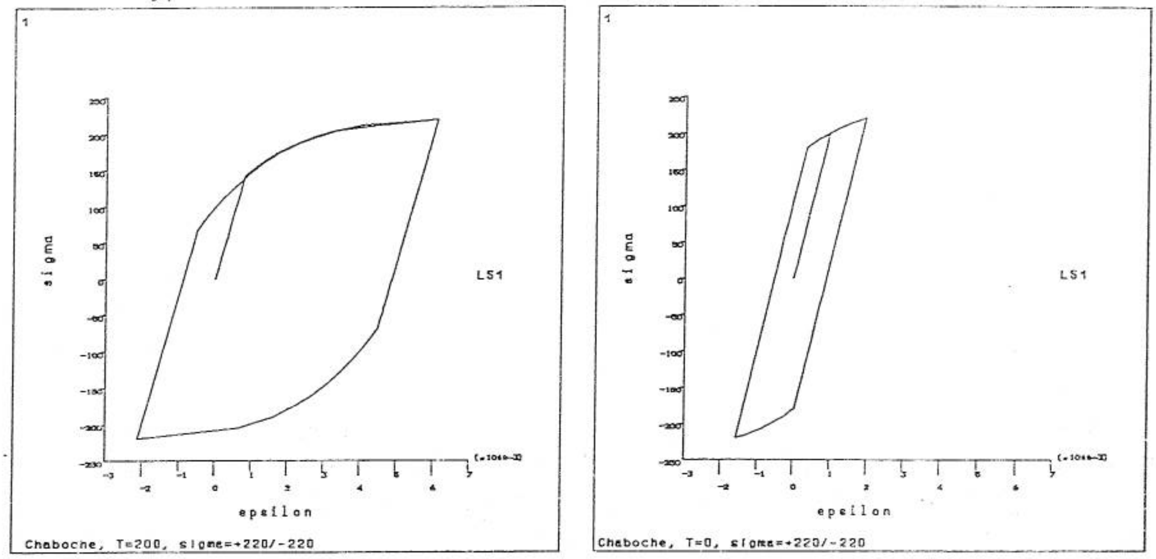

Fig. 11: Chaboche model, isothermal cycles, a) T=200, b) T=0

C, b, Q_0 and Q_M can be temperature-dependent. To obtain the independence of the temperature history

$$\alpha^* = \frac{\alpha}{C(T)} \tag{27}$$

is stored as a history variable instead of α which is recalculated in the next step by multiplying α^* by $C(T_{act})$. This is according to Chaboche[4], McDowell[6] and Ohno/Wang[10]. For R the proposal of Chaboche[4] is to store

$$r = \frac{R}{bQ} \tag{28}$$

but this does not lead to the wanted effect if both b and Q are temperature-dependent. Therefore, R is divided by the solution of the differential equation (17) for constant Q to obtain the state variable

$$r = \frac{R}{Q(1-e^{-bp})} \tag{29}$$

The non-isothermal cycles according to fig. 2 yield a large amount of increasing plasticity (fig 12a). Let us now consider the case T=200 with the stress range +220/ -180.4 (fig 12b). Increasing plasticity occurs in this isothermal case, too. The lower stress in this example is chosen such that the plastic strain is equivalent to that of the loading +220/T=200 and -220/T=0. That means the temperature-history independence holds, but the Chaboche model, at least with m=1, overpredicts ratchetting.

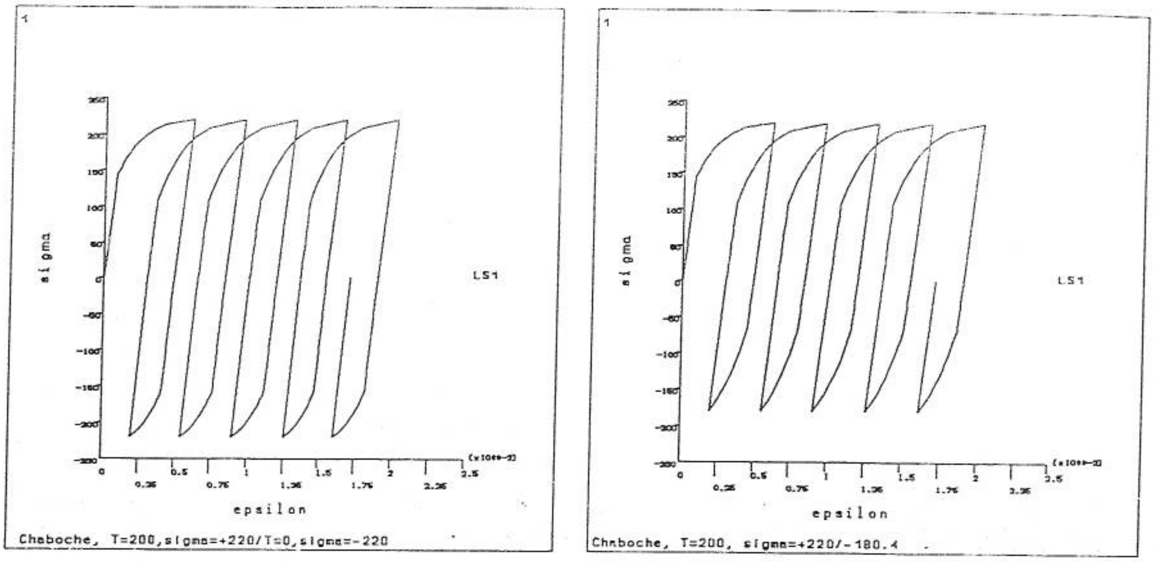

Fig. 12: a) non-isothermal cycles with mean stress 0
b) isothermal cycles with same plastic strain

Setting γ=0, μ=4 as well as b=100, Q_0=79, Q_M=0 for T=0 and b=151, Q_0=40, Q_M=0 for T=200 and applying the load and temperature history of fig. 1 one obtains the stress-strain curve of fig. 13b. The hysteresis loop slims because of the short-range terms, and then it is enlarged due to the long-range behaviour. It will come to a standstill when the saturation values of Q are reached.

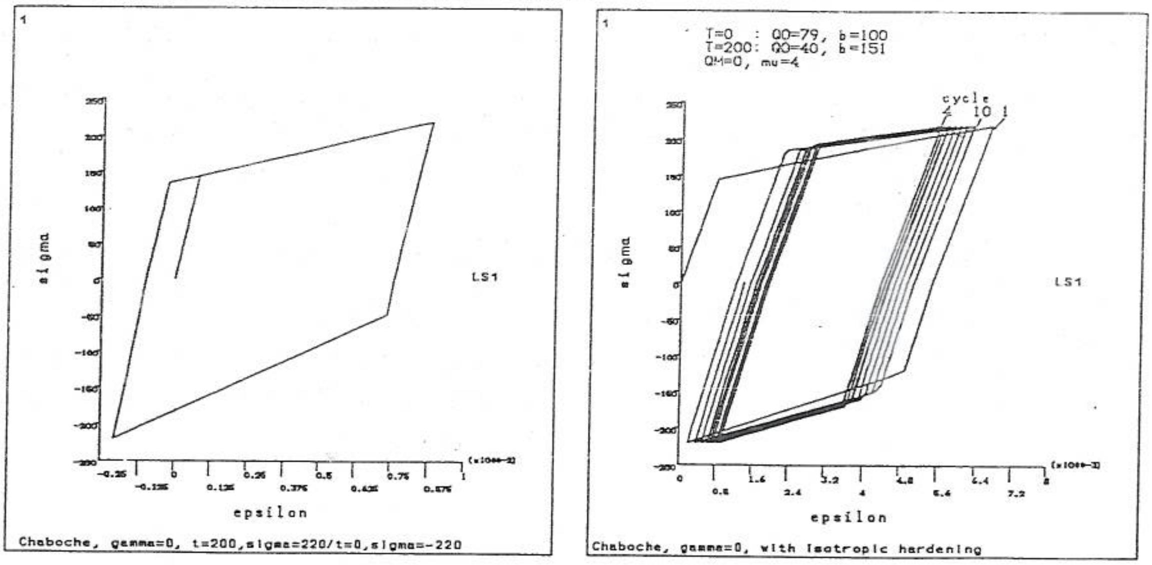

Fig. 13: Linear kinematic hardening, a) without b) with isotropic hardening

Extensions to the MKIN model

As shown in conjunction with the Chaboche model such effects concerning the form of the hysteresis loop (which can be observed in experiments) can be modeled by isotropic hardening. The following extension to MKIN is implemented in 1d at the moment. For MKIN a stress-strain curve is given. Here the user specifies which percentage β of the hardening in that curve is isotropic. The remaining part is kinematic. To include the long-range terms β depends on p

$$\beta = \beta_M + (\beta_0 - \beta_M) e^{-\delta p} \tag{30}$$

The results with this model (including eq. (14)) are very similar to that of the Chaboche model with linear kinematic hardening (γ=0, fig. 13) as far as they concern the position of the hysteresis loop, but here nonlinear hardening is included. That means the classical models are not out of date and have the advantage of comprehensible input data.

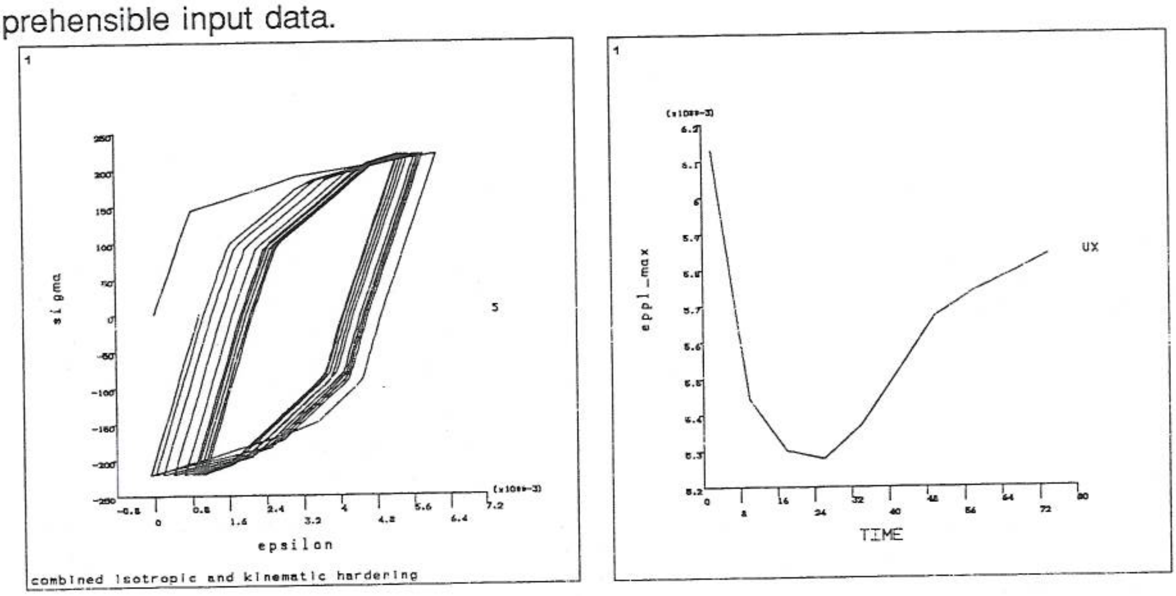

Fig. 14: MKIN extended by isotropic hardening
a) stress-strain curve for 10 non-isothermal cycles
b) maximum strain versus time

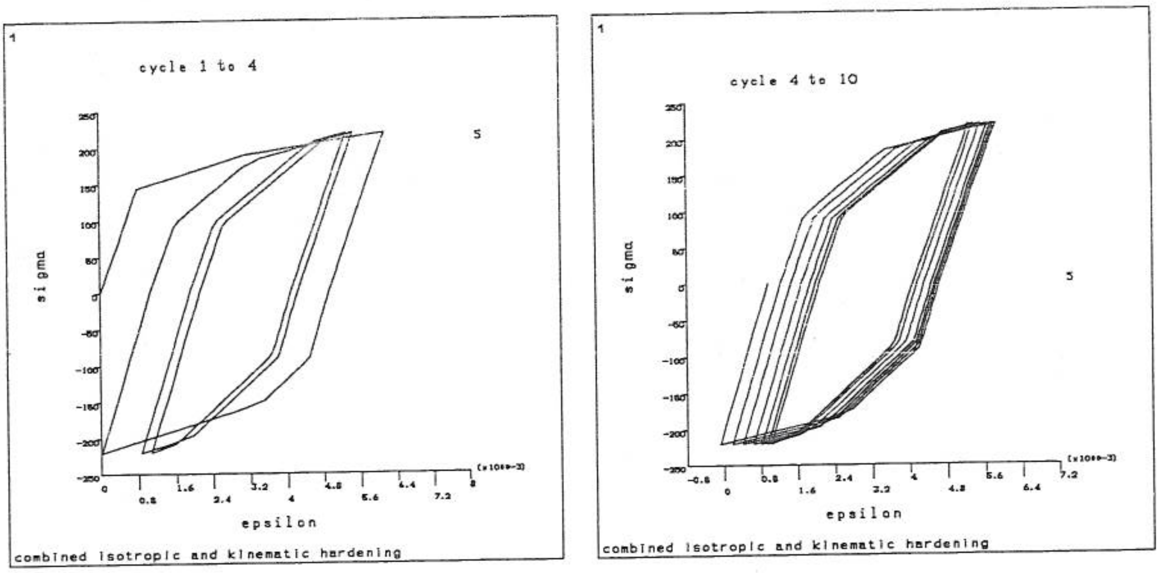

Fig. 15: extended MKIN, a) first cycles b) last cycles

Conclusions

The ANSYS plasticity models BKIN and MKIN are correct implementations of kinematic hardening in the isothermal case. Standard BKIN complies with the US-

Nuclear standard also in the non-isothermal case. So does option 1 which does not overpredict thermal ratchetting. Standard MKIN cannot be recommended for non-isothermal cyclic loading.
The simple modifications to MKIN outlined above should be made available to all users like the BKIN options. A combined isotropic-kinematic model should be offered. The Chaboche model - developped in cooperation of SASI and CAD-FEM - should be improved before being applied to real problems.

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CAD-FEM-Chaboche-Modell

Das Plastizitätsmodell in Anlehnung an Chaboche benutzt die Fließbedingung

$$J(\boldsymbol{\sigma}-\boldsymbol{\alpha}) = \sigma_F + R \tag{1}$$

wobei
- J die Von-Mises-Fließbedingung bedeutet,
- $\boldsymbol{\sigma}$ die Spannungskomponenten,
- $\boldsymbol{\alpha}$ die Verschiebung der Fließfläche (für kinematische Verfestigung),
- σ_F die Anfangsfließgrenze und
- R die Erhöhung der Fließgrenze infolge isotroper Verfestigung

$\boldsymbol{\alpha}$ und R sind Summen:

$$\alpha = \sum_{i=1}^{m} \alpha_i \, , \quad R = \sum_{i=1}^{m} R_i$$

Für die $\boldsymbol{\alpha}_i$ gilt die Evolutionsgleichung

$$d\boldsymbol{\alpha}_i = (C_i \, d\boldsymbol{\varepsilon}^{pl} - \gamma_i \boldsymbol{\alpha}_i \, d\varepsilon_{eq}^{pl})$$

Dabei sind
- $d(\ldots)$ Inkremente,
- C_i, γ_i Konstante,
- $\boldsymbol{\varepsilon}^{pl}$ die plastischen Dehnungskomponenten und
- $\varepsilon_{eq}{}^{pl}$ die kumulierte plastische Dehnung.

Für die R_i gilt

$$dR_i = b_i(Q_i - R_i) d\varepsilon_{eq}^{pl}$$

Dabei sind b_i und Q_i Materialkonstanten.

In der CAD-FEM-Implementierung ist m auf 5 beschränkt.

C_i und Q_i sind temperaturabhängig vorgesehen; Werte für bis zu fünf Temperaturen können eingegeben werden. Um Temperaturratcheting zu vermeiden, wird statt α

$$\alpha_i^* = \int \frac{d\alpha_i}{C_i(T)} \quad und \quad R_i^* = \int \frac{dR}{Q_i(T)}$$

gespeichert und in (1)

$$\alpha_i = C_i(T)\alpha_i^* \quad und \quad R_i = Q_i(T)R_i^*$$

verwendet.

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CAD-FEM-Mroz-Modell

Das Mroz-Modell für Plastizität mit kinematischer Verfestigung ist mit dem in ANSYS standardmäßig vorhandenen Besseling-Modell (MKIN) verwandt, unterscheidet sich jedoch davon in zwei Punkten:

1) Die Eingabe läßt wie MKIN je fünf Spannungs-Dehnungs-Paare bei fünf verschiedenen Temperaturen, jedoch mit unterschiedlichen Dehnungsstützstellen zu.
2) Die Verschiebung der Teilfließflächen wird so gewählt, daß sie sich nicht schneiden können. Das ergibt bei nichtproportionaler Belastung einen Unterschied im Ergebnis.

Ähnlich MKIN, Option 2, (s. [1]) wird eine Änderung der Verschiebung der Fließflächen mit der Temperaturänderung berücksichtigt.

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